Normally, you can easily determine whether or not a mathematical statement is true or false by looking at it and performing calculations to simplify the statement. However, to prove mathematical statements are true, you can use properties of numbers to justify each step of the simplification of the statement.

**Using Justifications with the Order of Operations**

When you simplify an expression, you can use an arithmetic property to justify each step. Common properties are listed below:

- **Commutative Property of Addition**
  \[ a + b = b + a \]
- **Associative Property of Addition**
  \[ (a + b) + c = a + (b + c) \]
- **Commutative Property of Multiplication**
  \[ ab = ba \]
- **Associative Property of Multiplication**
  \[ (ab)c = (ac)b \]
- **Distributive Property**
  \[ a(b + c) = ab + ac \]
- **Addition Property of Opposite**
  \[ a + (-a) = 0 \]

**EXAMPLE 1**

Show that \(5(8 \cdot 4) + 11 + 2\left(\frac{5}{2} \cdot 8\right) + \frac{1}{3}(15 - 33) = 205\)

Simplify the left side of the equation. Use a property to justify each step.

\[
5(8 \cdot 4) + 11 + 2\left(\frac{5}{2} \cdot 8\right) + \frac{1}{3}(15 - 33) \\
= 5(4 \cdot 8) + 11 + 2\left(\frac{5}{2} \cdot 8\right) + \frac{1}{3}(15 - 33) \quad \text{Commutative Property of Multiplication} \\
= (5 \cdot 4) \cdot 8 + 11 + 2\left(\frac{5}{2} \cdot 8\right) + \frac{1}{3}(15 - 33) \quad \text{Associative Property of Multiplication} \\
= 160 + 11 + \left(2 \frac{5}{2} \cdot 8\right) + \frac{1}{3}(15 - 33) \quad \text{Associative Property of Multiplication} \\
= 160 + 11 + 40 + 5 - 11 \quad \text{Distributive Property} \\
= 160 + 40 + 11 + 5 - 11 \quad \text{Commutative Property of Addition} \\
= 200 + 11 - 11 + 5 \quad \text{Commutative Property of Addition} \\
= 200 + 5 \quad \text{Addition Property of Opposites} \\
= 205
\]

**CA Standards Check 1**

1a. Show that \(2(10 \cdot 14) + 16 + \frac{1}{4}(8 \cdot 10) - 16 = 300\)

1b. Show that \(4\left(\frac{22}{4} - 1\right) \cdot 3 \cdot \frac{15}{6} = 135\)
Using Justifications to Show Equal Expressions

In order to show that two expressions are equal, make one of the expressions match the second. A complete match will have the same terms in the same order. It is important to only complete one step of the manipulation of one of the sides with each step, so that a property of numbers can be used in the justification.

**EXAMPLE 2**

Show that $5(2 + x) + 12x = 17x + 10$

*Simplify the left side of the equation. Use a property to justify each step.*

1. $5(2 + x) + 12x$
2. $(10 + 5x) + 12x$  Distributive Property
3. $10 + 5x + 12x$  Associative Property of Addition
4. $10 + 17x$  Simplify
5. $17x + 10$  Commutative Property

**CA Standards Check 2**

2a. Show that $4y - 8(1 - y) = 12y - 8$

2b. Show that $3x + 11 - x(2x - 1) = 11 + 4x - 2x^2$
1. What property justifies stating that $3 + 10 + 2 = 3 + 2 + 10$?
   - A. Commutative Property of Addition
   - B. Associative Property of Addition
   - C. Addition Property of Opposite
   - D. Distributive Property

2. What two properties justify that $4 + 23 - 4 = 23$?
   - A. Distributive Property and Addition Property of Opposite
   - B. Commutative and Associative Properties of Addition
   - C. Commutative Property of Addition only
   - D. Commutative Property of Addition and Addition Property of Opposites

3. What two properties justify that $2(x + 4) - 8 = 2x$?
   - A. Distributive Property and Associative Property of Addition
   - B. Commutative and Associative Properties of Addition
   - C. Distributive Property and Addition Property of Opposite
   - D. Commutative Property of Addition and Addition Property of Opposites

4. Which property states you can change the order of two numbers and not affect their sum?
   - A. Addition Property of Opposites
   - B. Associative Property of Addition
   - C. Commutative Property of Addition
   - D. Addition Property of Equality

5. What property justifies stating that $\frac{7(9 \cdot 4)}{3} = 21 \cdot 4$?
   - A. Commutative Property of Multiplication
   - B. Associative Property of Multiplication
   - C. Addition Property of Opposite
   - D. Distributive Property

6. What two properties justify that $8 + (-2 \cdot 6) + \frac{48}{6} + (-8) = -16$?
   - A. Associative Property of Multiplication and Commutative Property of Addition
   - B. Commutative Property of Addition and Addition Property of Opposites
   - C. Distributive Property and Associative Property of Multiplication
   - D. Commutative Property of Multiplication and Addition Property of Opposites